QI Macros Histogram: Formulas and Calculations

The purpose of this document is to provide detailed information on the formulas in the QI Macros. This paper provides the formulas, a sample calculation and a histogram run in the QI Macros using the same data provided.

Formulas for Cp and Cpk

$$Cp = \frac{(USL - LSL)}{6\hat{\sigma}}$$

$$CpU = \frac{(USL - \overline{X})}{3\hat{\sigma}}$$

$$CpL = \frac{(\overline{X} - LSL)}{3\hat{\sigma}}$$

$$Cpk = Min(CpU, CpL)$$

Formulas for Pp and Ppk

For Process Performance, use standard deviation (σ) of the population instead of sigma estimator:

$$Pp = \frac{(USL - LSL)}{6\sigma}$$

$$PpU = \frac{(USL - \overline{X})}{3\sigma}$$

$$PpL = \frac{(\overline{X} - LSL)}{3\sigma}$$

$$Ppk = Min(PpU, PpL)$$

Another View of the Formulas

Formulas for Cp and Pp

Cp	Pp
$\frac{(\text{USL-LSL})}{(6 * \text{sigma estimator})}$	$\frac{(\text{USL-LSL})}{(6 * \text{standard deviation})}$
Use when you have a sample	Use when you have the total population

Formulas for Cpk and Ppk

	Cpk		Ppk
	Minimum of CpU and CpL		Minimum of PpU and PpL
CpU	$\frac{(\text{USL-Xbar})}{(3 * \text{sigma estimator})}$	PpU	$\frac{(\text{USL-Xbar})}{(3 * \text{standard deviation})}$
CpL	$\frac{(\text{Xbar - LSL})}{(3 * \text{sigma estimator})}$	PpL	$\frac{(\text{Xbar - LSL})}{(3 * \text{standard deviation})}$
	Use when you have a sample		Use when you have the total population

Points to note:

- **Xbar** = the average of the data points = $\sum X / n$
- Changing the spec limits, will change Cp and Pp and may change Cpk and Ppk
- Cp and Cpk use sigma estimator because they assume your data represents a sample of the population
- Pp and Ppk use standard deviation because they assume your data represents the total population

Formula for Sigma Estimator

Standard deviation of a population can be estimated from the average range or average standard deviation of the samples in each subgroup. These are used to calculate C_p and $C_{pk.}$

$$\hat{\sigma} = \frac{\overline{R}}{d_2}$$

$$\hat{\sigma} = \frac{\overline{s}}{c_4}$$

For n=1-4, use R/d_2 formula; for n>4 use s/c_4 formula

Here is Another Way of Looking at It

Subgroup Size	Sigma Estimator Formula	Definitions
1 to 4	$\frac{(\text{Rbar})}{(d_2)}$	**Rbar** = Average of the ranges **d_2** is a constant based on the sample size
5 or more	$\frac{(\text{Sbar})}{(c_4)}$	**Sbar** = Average of the standard deviations **c_4** is a constant based on the sample size

Constants for Sigma Estimator Calculation (Source:ASTM Manual on Presentation of Data and Control Chart Analysis – Table 16, 2002)

Subgroup Size	Constant	Value	Subgroup Size	Constant	Value
1	d_2	1.128	**26**	c_4	0.9901
2	d_2	1.128	**27**	c_4	0.9905
3	d_2	1.693	**28**	c_4	0.9908
4	d_2	2.059	**29**	c_4	0.9912
5	c_4	0.94	**30**	c_4	0.9915
6	c_4	0.9515	**31**	c_4	0.9917
7	c_4	0.9594	**32**	c_4	0.992
8	c_4	0.965	**33**	c_4	0.9922
9	c_4	0.9693	**34**	c_4	0.9925
10	c_4	0.9727	**35**	c_4	0.9927
11	c_4	0.9754	**36**	c_4	0.9929
12	c_4	0.9776	**37**	c_4	0.9931
13	c_4	0.9794	**38**	c_4	0.9933
14	c_4	0.981	**39**	c_4	0.9935

15	c_4	0.9823	**40**	c_4	0.9936
16	c_4	0.9835	**41**	c_4	0.9938
17	c_4	0.9845	**42**	c_4	0.9939
18	c_4	0.9854	**43**	c_4	0.9941
19	c_4	0.9862	**44**	c_4	0.9942
20	c_4	0.9869	**45**	c_4	0.9944
21	c_4	0.9876	**46**	c_4	0.9945
22	c_4	0.9882	**47**	c_4	0.9946
23	c_4	0.9887	**48**	c_4	0.9947
24	c_4	0.9892	**49**	c_4	0.9948
25	c_4	0.9896	**50**	c_4	0.9949

Formulas for One Sided Spec Limits

LSL Only	USL Only
Cp = Cpk = CpL **Pp = Ppk = PpL**	**Cp = Cpk = CpU** **Pp = Ppk = PpU**

Formula for Defects in Parts Per Million

Actual	Estimated for Population
$\frac{\text{(\# of non conforming)*1000000}}{\text{(\# of parts)}}$	PPMU = NORMSDIST(Z upper)*1000000 + PPML = NORMSDIST(Z lower)*1000000

Formulas for Z Scores

Z scores help estimate the non-conforming PPM. Z scores *standardize* +/-3*stdev values into +/-3.

Zlower	(Xbar-LSL)/stdev
Zupper	(USL-Xbar)/stdev
Zbench is the Z score for the Expected PPM	normsinv(1-(Expected PPM/1,000,000))
ZT (target) = Cpk for a target value instead of the USL or LSL. If not defined, use the midpoint between the USL and LSL	(Xbar-Target)/(3*sigest)

Sample Calculation

Let's perform calculations using the following sample data from Montgomery, Intro to SPC, 4th Ed., pgs. 353-358. You can download this data as part of the QI Macros test data at http://www.qimacros.com/testdata/SPCManufacturing.xls Open the spreadsheet and click on the histogram tab.

Sample	Obs 1	Obs 2	Obs 3	Obs 4	Obs 5
S1	265	205	263	307	220
S2	268	260	234	299	215
S3	197	286	274	243	231
S4	267	281	265	214	318
S5	346	317	242	258	276
S6	300	208	187	264	271
S7	280	242	260	321	228
S8	250	299	258	267	293
S9	265	254	281	294	223
S10	260	308	265	283	277
S11	200	235	246	328	296
S12	276	264	269	235	290
S13	221	176	258	263	231
S14	334	280	265	272	283
S15	265	262	271	245	301
S16	280	274	253	287	258
S17	261	248	260	274	337
S18	250	278	254	274	275
S19	278	250	265	270	298
S20	257	210	280	269	251

- Assume the USL = 346 and the LSL = 200.
- Since there are 5 subgroups, sigma estimator will use the formula SBar/c_4
- If we look in the table above, the constant for a subgroup of 5 is 0.94
- Other calculations for this data set are:
  - Xbar = 26,446/100=264.46
  - Standard deviation = 31.85
- If you are trying to recalculate this manually, use the statistical functions in Excel to calculate: standard deviation, normdist and normsinv.

Calculations

	Formula	Calc	Calc	Value
Cp	$\frac{(\text{USL-LSL})}{6*(\text{SBar}/c_4)}$	$\frac{(346-200)}{6*(30.02/.94)}$	$\frac{146}{191.62}$	.762
Pp	$\frac{(\text{USL-LSL})}{(6*\text{standard dev})}$	$\frac{(346-200)}{(6*31.85)}$	$\frac{146}{191.10}$	.764

CpU	(USL-Xbar) / 3 *(SBar/c_4)	(346-264.46) / (3 * (30.02/.94)	81.54 / 95.81	.851
CpL	(Xbar - LSL) / 3 * (SBar/c_4)	(264.46-200) / (3 * (30.02/.94)	64.46 / 95.81	.673
Cpk	Minimum of CpU and CpL	.851 vs .673		.673
PpU	(USL-Xbar) / (3 * standard deviation)	(346-264.46) / 3 * 31.85	81.54 / 95.55	.853
PpL	(Xbar - LSL-) / (3 * standard deviation)	(264.46-200) / 3 * 31.85	64.46 / 95.55	.675
Ppk	Minimum of PpU and PpL	.853 vs .675		.675
Actual PPM	(# of non conforming parts / (# of parts)	3 / 100	30,000 / 1,000,000	30,000
Est PPM	PPM Upper + PPM lower = NORMDIST(Zu) * 1,000,000 + NORMDIST(Zl) * 1,000,000	NORMDIST (2.5601) * 1000000 + NORMDIST (2.0238) * 1000000	21482.34 + 5228.07	26,710.41
Z upper (Zu)	(USL-Ave) / standard deviation	346 - 264.46 / 31.85	2.5601	
Z lower (Zl)	(Ave-LSL) / standard deviation	264.46 - 200 / 31.85	2.0238	
Z bench	normsinv(1-(Expected PPM/1,000,000))	normsinv(1-(.0267104)	normsinv(.97329)	1.93
Target	Defined by Customer or Midpoint between	(346 +200) / 2	546 / 2	273

	USL and LSL			
Z target	ABS(Xbar-Target) / (3*sigest)	264.46 - 273 / 3*(30.02/.94)	8.54 / 95.81	.09

The final calculated amounts are:

Cp	.762
Cpk	.673
Pp	.764
Ppk	.675
PPM	30,000
Est PPM	26,710.41
Zbench	1.93
Z target	.09

Since the standard deviation and sigma estimator were fairly close in value, the Cp and Pp and Cpk and Ppk values were very similar.

Histogram in the QI Macros SPC Software

If you run a histogram in the QI Macros using the above data you should get the same results. Make sure you input the USL= 346 and the LSL = 200.